Mathematical configuration of real physical space

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Abstract

Real physical space, as a purely mathematical structure formed according to the rules of set theory, topology, and fractal geometry, was proposed by Michel Bounias (1943–2003) and the author. It emerges as a mathematical lattice of primary topological balls, which was named a tessellattice, and the size of a cell/ball in the tessellattice is comparative with the Planck length, $\sim 10^{-35}$ m. Discrete fractal properties of the tessellattice allow the prediction of scales at which submicroscopic to cosmic structures should occur. This approach allows the development of a submicroscopic concept of physics, which describes Nature at a much deeper level than offered by the quantum-mechanical formalism developed at the atom scale, $\sim 10^{-10}$ m. In addition, the approach makes it possible to define such fundamental physical notions as mass and charge from first submicroscopic principles, and this actually means that fundamental mathematics lays down the basic concepts of physics.

Key words: topology, set theory, fractals, real physical space, quantum mechanics

1. Introduction

As known, in classical mechanics, the state of a particle is precisely defined, and all observable quantities have exact values. Although in the gravitation physics an important role plays spacetime, which in the formalism of general relativity is a smooth manifold with a pseudo-Riemannian metric $ds^2 = \sum_{\alpha\beta} g^{\alpha\beta} dx^\alpha dx^\beta$ of signature $(+,-,-,-)$, so gravitation was reduced to the examination of the curvature of spacetime.

In the quantum mechanical formalism we meet a wave-particle duality, wavefunction ψ, and the quantum system is described mathematically by using a complex separable Hilbert space. Particle physics is based on the strong and weak interactions, quantum field theory, and quantum chromodynamics (the latter also largely covering the physics of nuclear interactions). In addition, particle physics relies heavily on the mathematical apparatus of symmetry groups.

From this we can see that each of the named branches of physics is practically unrelated to the other, however, theoretical particle physicists have lumped them all together...

Physicists perceive topology as a mathematical discipline that studies shapes and their arrangement in space and it is applicable for the evaluation of properties of systems, in particular, geometrical objects that remain unchanged as the system is continuously bent, twisted, or otherwise deformed. Topology has come to be recognized as being of an indispensable tool in particle physics, solid state physics and optics [1]. In particle physics, researchers construct topological models of particle-like continuous fields, study topological effects in quantum field theories focusing on phenomenological applications in which abstract objects (like the Higgs particle with a certain symmetry) are endowed with a peculiar topology [2].

A valid question arises: Is it possible to link the various named branches of physics into one single structure, which will also be consistent with topology? Perhaps the answer will be positive if we first construct physical space, and only then derive physics from it. But physical space should be inextricably linked with topology that studies the behaviour of spatial objects. So, in this case, we will be able to solve another problem – to derive physics from space, taking into account its topology.

In his recent work, Maudlin [3] has developed a theory of linear structures (as collection of lines or directed lines), which in his opinion should provide a new mathematical tool for understanding geometrical structure, which so far has been described by the open set structure of a physical space that is determined by a collection of open sets.

Nevertheless, much deeper views on the geometry of the universe were expressed early by Mortensen and Nerlich [4]. Their approach does not deny sets and a physical interval has been considered to be a whole whose point-parts are all the members of a set which is the image under the one-one correspondence of some real number interval. Density of an interval is straightforward: between any two points of the interval is a third. The universe is dense iff every interval is dense.

They show that every set, which can be made up from existing objects, exists and hence we need some explanation for just why these objects are. They tried to contemplate how matter could continuously occupy space in which a piece of matter is continuous and connected because it is made up of points of matter, and there is a continuous and connected physical interval such that every matter-point is at some spatial point of the interval, and every point of the interval has a point of matter at it. Thereby, physical topology was treated as the relatedness of points, not the existence of certain sets of ordered pairs and hence the topological properties of a matter were inspected as derivative from the topological properties of the space in which the matter is embedded.

Research of the constitution of real physical space carried out by Michel Bounias and the author [5–9], made it possible to completely replace the vague concept of the physical vacuum (and the mechanical ether preceding it) with a certain primary substrate from which particles arise and which has a clear mathematical structure and is determined by explicit fundamental laws of Nature. The main mathematical disciplines laid as the basis of the research were set theory, topology and fractal geometry.

This paper discusses topological features of ordinary physical space as they arise from pure mathematical constructions. There is no doubt that the proposed approach is correct, as it enables the direct derivation of the Standard Model of particle physics (with the necessary corrections) and quantum mechanics from truly first principles. Moreover, the approach was experimentally verified [9] and even several technologies were implemented (see, e.g. Refs. [10, 11]).

2. Main concepts

First of all, we have to recall that an elementary particle can be created in any point of the universe. At the same time all such points take part in the formation of distances between physical objects. Therefore, this denotes that physical space is made of {objects + distances} and all comes from the same origin: manifold of sets. Hence, the association of discrete sets whose interior is continuous although covered by discrete subparts (micro), as derived from the empty set $\emptyset$ provides a wonderfully organized fundamental 'substrate', i.e., a mathematical lattice.

Let us now consider the notions of ***measure*** and ***distances*** in a broad topological sense that includes also the assessment of the dimensionality of a space.

2.1. Measure

Usually, a measure is a comparison of the measured object with some unit taken as a standard, nevertheless, sets or spaces and functions are measurable under other conditions.

A mapping f of a set E into a topological space T is measurable if the reciprocal image of an open of T by f is measurable in E. And a set measure on E is a mapping m of a tribe B of sets of E

in the interval $[0,\ \infty]$, exhibiting denumerable additivity for any sequence of disjoint subsets (b_n) of B, and denumerable finiteness, i.e., the following correspondence must be fulfilled:

$$\mathcal{m}\left(\cup_{n=0}^{\infty} b_n\right) = \cup_{n=0}^{\infty} \mathcal{m}(b_n) \tag{1}$$

where $\exists\, b_n,\ b_n \in B,\ E = \cup\, b_n,\ \forall \in$ N, $\mathcal{m}(b_n)$ are finite.

In such a case a unit of measure that plays the role of a standard is the part subordinated to a gauge (J). Usually a gauge having non-zero real values is a function defined on all bound sets of the considered space and following Tricot [12] it has the subsequent properties:

- a singleton has measure naught: $\forall x, J(\{x\}) = 0$;
- (J) is continued with respect to the Hausdorff distance;
- (J) is growing: $E \subset F \Longrightarrow J(E) \subset J(F)$;
- (J) is linear: $F(r \cdot E) : r \cdot J(E)$.

Therefore, in topology the concept of distance is defined: usually, a diameter/size or a deviation are used and such distances one can apply on fully ordered sets.

The Jordan and Lebesgue measures demand respective mappings (I) and (m^*) on spaces that must be provided with $\cap$, $\cup$ and $\complement$. Bounias and Bonaly [13] showed that in spaces of the $\mathbb{R}^n$ type, in addition to the mentioned rules a tessellation of balls should be involved. This means that a distance to be available for the measure of diameters of intervals.

A set of measure naught was defined by Borel (1912) first as a linear set (E); all points of E are contained in intervals whose sum is lower than (e) (some more detail see in Ref. [5]).

2.1. Distances

Now we can consider distances. Following Borel, the length of an interval $F = [a, b]$ is

$$L(F) = (a - b) - \sum_n L(C_n) \tag{2}$$

where C_n are the open intervals in the fundamental segment.

The distance (2) is required in the Hausdorff distances of sets (E) and (F). Let E(e) and F(e) are the covers of E or F by balls $B(x, e)$, respectively, where $x \in E$ or $x \in F$. So, the distance is:

$$\mathrm{dist_H}(E, F) = \inf\{e :\ E \subset F(e)\ \wedge\ F \subset E(e)\}, \tag{3}$$

$$\mathrm{dist_H}(E, F) = (x\ \subset\ E,\ y \subset F : \inf \mathrm{dist}(x, y\ )). \tag{4}$$

Such a distance is not necessarily compatible with topological properties of the concerned spaces, nevertheless, the intervals can be replaced by topological balls. A more general approach may involve a path $\varphi(x, y)$.

Bounias [13–18] proved that in a partly ordered space, for sets A and B the symmetric difference $\Delta(A, B) = \complement_{A \cup B}(A \cap B)$ to be a true distance that is holding for more than two sets. If $A \cup B = \emptyset$, the distance remains $\Delta = A \cup B$. If $A \cup B = \emptyset$ and $(A, B) \subset E$, the distance remains $\Delta = A \cup B$.

In a general case, a distance between spaces A, B within their common embedding space E is issued by the intersection of a path-set $\varphi(A, B)$ joining members of A to members of B with the complementary $\complement_{A \cup B}$, such that the path $\varphi(A, B)$ is a continued sequence of a function f of a gauge (J) belonging to the ultrafilter of topologies on $\{E, A, B, \dots\}$.

The path $\varphi(A, B)$ is a set composed as $\varphi(A, B) = \cup_{a \in A,\ b \in B}\ \varphi(A, B)$ and they are defined in a sequence interval $[0,\ f^n(x)],\ x \in E$.

For any closed D situated between A and B, $f^n(m)$ intersects the frontiers of B, D and A, and the sequence f^n has points identified with b, $(d_i, d_j, \dots \in \partial D)$, and a. This means that the relative distance of A and B in E, noted $\Lambda_E(A, B)$ is contained in $\varphi(A, B)$:

$$\Lambda_E(A, B) \subseteq \varphi(A, B). \tag{5}$$

In the case of completely ordered space, the distance (d) between A and B is represented by the relation

$$d(A, B) \subseteq \mathrm{dist}(\inf A, \inf B) \cap \mathrm{dist}(\sup A, \sup B) \tag{6}$$

with the distance evaluated through either classical forms or even the set-distance $\Delta(A, B)$.

The set-distance is the symmetric difference between sets and that it can be extended to manifolds of sets and it possesses all properties of a true distance [15, 16]. In a topologically closed space, such distances are the open complementary of closed intersections called “instances” by Bounias. The intersection of closed sets is closed and the intersection of sets with nonequal dimensions is always closed [13], therefore, the instances stands for closed structures. This reflects physical-like properties, i.e. they characterise objects, and the distances as being their complementaries constitute the alternative class. Thus, a physical-like topological space tends globally to be subdivided into objects and distances as full components.

These properties state important

Bounias’ theorem [5]: Any topological space is metrizable as provided with the set-distance (Δ) as a natural metrics. All topological spaces are kinds of metric spaces called “delta-metric spaces”.

So, a distance $\Delta(A,B)$ is a kind of an intrinsic case $[\Lambda_{(A,B)}(A,B)]$ of $\Lambda_E(A,B)$ while $\Lambda_E(A,B)$ is called a "separating distance". The separating distance also stands for a topological metrics. Hence, if a physical space is a topological space, it will always be measurable.

2.3. Space dimensions

If the structure of members of a set is unknown, a problem arises how to distinguish unordered N-tuples and ordered N-tuples. The problem is important for the assessment of the actual dimension of a space.

Let a fundamental segment (AB) has intervals $L_i = \left[A_i, A_{(i+1)}\right]$, a generator is composed of the union of several such intervals $G = \cup_{(i\in[1,n])} L_i$ and the similarity coefficients be defined for each interval by $\varrho_i = \mathrm{dist}(A_i, A_{(i+1)}/\mathrm{dist}(AB)$.

The similarity exponent of Bouligand, e, is such that for a generator with n parts

$$\sum_{i\in[1,n]} (\varrho_i)^{\mathrm{e}} = 1 \tag{7}$$

When all intervals have (at least nearly) the same size, then the various dimension approaches according to Bouligand, Minkowski, Hausdorff and Besicovitch are reflected in the resulting relation

$$n \cdot (\varrho)^{\mathrm{e}} = 1, \tag{8}$$

that is:

$$\mathrm{e} \approx -\,\mathrm{Log}\, n / \mathrm{Log}\, \varrho \tag{9}$$

where $\varrho < 1$. When e is an integer, it reflects a topological dimension showing that a fundamental space E can be tessellated with an entire number of identical balls B exhibiting a similarity with E, upon coefficient ϱ.

In a space composed of members identified with some abstract components, it may not be found tessellating balls all having identical diameter. Then a measure should be used as a probe for the evaluation of the coefficient of size ratio ϱ needed for the calculation of a dimension.

Fig. 1 shows the principle of the dimensionality formation of a simplex. A 3-object has dimension 2 iff the longer side of given $A^1_{\max}$ fulfils the condition, for the triangular strict inequality, where M denotes an appropriate measure:

$$M(A^1_{\max}) \; < \; M(A^1_2) \; < M(A^1_3) \tag{10}$$

Generally for a space X being a N-object

$$M(A_{\max}^{k}) < \cup_{i=1}^{N-1} \{M(A_i^k)\} \quad (11)$$

where N is the number of vertices, i.e. members in X, $k = (d - 1 = N - 2)$, and $A_{\max}^{k}$ is the k-face with maximum size in X.

According to the relation (11), for a 2-object, i.e. when $N = 2$, $X = \{x_1, x_2\}$ we have the dimension equals 1 iff $x_1 < (x_1 + x_2)$, i.e. iff $x_1 < x_2$ (Fig. 1). This qualifies the lower state of an existing space X^1.

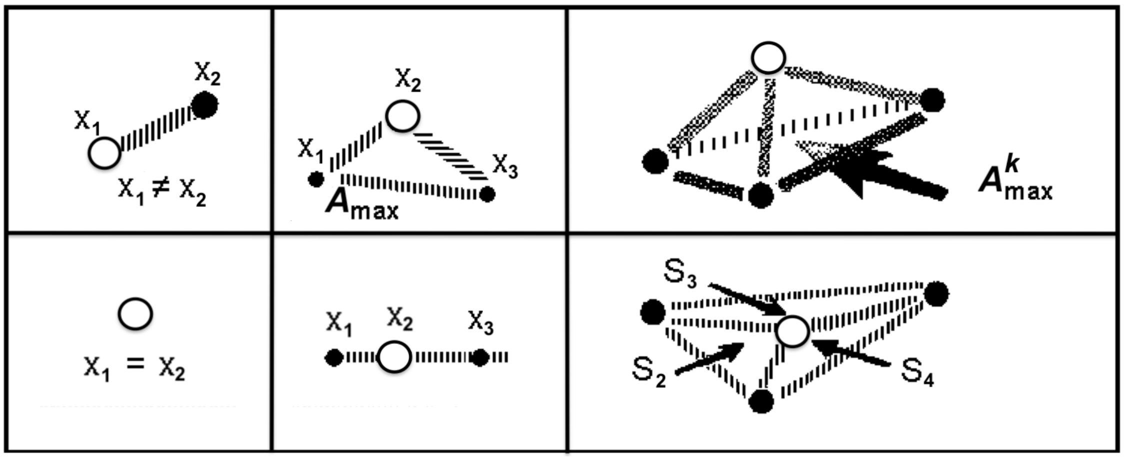

Figure 1. The first three steps of the N-angular strict inequality for the assessment of the dimensionality of a simplex. In the lower right picture, the larger side standing for $A_{\max}^{k}$ is S_1, such that in a 2D space we have exactly: $S_1 = S_2 + S_3 + S_4$.

Now if X is decomposed into the union of balls represented by D-faces A^{D} having dimension Dim $(A^{\mathrm{D}}) = \mathrm{D}$ by the relation (11) and size $M(A^1)$ for a 1-face. Such a D-face is a D-simplex S_j whose size, as a ball, is evaluated by $M(A_{\max}^{1})^{\mathrm{D}} = S_j^{\mathrm{D}}$. Let $\mathcal{N}$ is the number of such balls that can be filled in a space H, so that

$$\cup_{i=1}^{\mathcal{N}} \{S_j^{\mathrm{D}}\} \subseteq (H \approx L_{\max}^{\mathrm{d}}) \quad (12)$$

where H is the ball whose size is evaluated by L^{d}, L is the size of a 1-face of H, and d is the dimension of H. If $\forall S_j$, $S_j \approx S_0$, then the dimension of H is

$$\mathrm{Dim}(H) \approx (\mathrm{D} \cdot \mathrm{Log}\, S_0 + \mathrm{Log}\, \mathcal{N})/\mathrm{Log}\, L_{\max}^{1}. \quad (13)$$

The relation (13) stands for an interior measure in the Jordan's sense.

In contrast, if we assume that the reunion of balls covers the space H, then $\mathrm{Dim}(H)$ will rather represents the capacity dimension, which remains an evaluation of a fractal property.

3. The founding element and founding lattice

As was shown by Bounias and Bounaly [15, 16], the existence of the empty set is a necessary and sufficient condition for the existence of abstract mathematical spaces (W^n) endowed with topological dimensions (n). That is, the empty set appears as a set without members though containing empty parts. The empty set exhibits (i) self-similarity at all scales and (ii) nowhere derivability, i.e. two characteristics of fractal structures.

The general postulate that some set exists was reduced [15, 16] to a weaker form, namely, to the axiom of the existence of the empty set. It was shown that providing the empty set $(\emptyset)$ with $(\in, \subset)$ as the combination rules (the same with the property of complementarity $(\complement)$) resulted in the definition of a magma allowing a consistent application of the first De Morgan's law without violating the axiom of foundation iff the empty set seen as a hyperset that is a nonwellfounded set. These results led to the formulation of an important theorem established by means of several lemmas, which is stated below.

Bounias' Theorem [5]. The magma $\emptyset^{\emptyset} = \{\emptyset, \complement\}$ constructed with the empty hyperset and the axiom of availability is a fractal lattice.

Remark. A magma is a set equipped with a single binary operation that must be closed by definition. Writing $(\emptyset^{\emptyset})$ denotes that the magma reflects the set of all self-mappings of $\emptyset$, which emphasizes the forthcoming results.

Lemma 1. The space constructed with the empty set cells of $E_{\emptyset}$ is a Boolean lattice.

Proof. (i) Let $\cup(\emptyset) = S$ denote a simple partition of $(\emptyset)$. Suppose that there exists an object (ε) included in a part of S, then necessarily $(\varepsilon) = \emptyset$ and its belongs to the partition.

(ii) Let $P = \{\emptyset, \emptyset\}$ denote a part bounded by $\sup P = S$ and $\inf P = \{\emptyset\}$. The combination rules $\cup$ and $\cap$ provided with commutativity, associativity and absorption are holding. In effect: $\emptyset \cup \emptyset = \emptyset$, $\emptyset \cap \emptyset = \emptyset$ and thus necessarily $\emptyset \cup (\emptyset \cap \emptyset) = \emptyset$, $\emptyset \cup (\emptyset \cup \emptyset) = \emptyset$. Thus, space $\{P(\emptyset), (\cup, \cap)\}$ is a lattice.

The null member is Ø and the universal member is $2^{\emptyset}$ that should be denoted by $\aleph_{\emptyset}$. Since in addition, by founding property $\complement_{\emptyset}(\emptyset) = \emptyset$, and the space of $(\emptyset)$ is distributive, then $S(\emptyset)$ is a Boolean lattice.

Lemma 2. $S(\emptyset)$ is provided with a topology of discrete space.

Proof. (i) The lattice $S(\emptyset)$) owns a topology. In effect, it is stable upon union and finite intersection, and its contains $(\emptyset)$.

(ii) Let $S(\emptyset)$ denote a set of closed units. Two units $\emptyset_1,\ \emptyset_2$ separated by a unit $\emptyset_3$ compose a part $\{\emptyset_1, \emptyset_2,\ \emptyset_3\}$. Then, owing to the fact that the complementary of a closed is an open: $\complement_{\emptyset_1,\emptyset_2,\emptyset_3}\{\emptyset_1, \emptyset_3\} = \emptyset_2$ and $\emptyset_2$ is open. Thus, by recurrence, $\{\emptyset_1, \emptyset_3\}$ are surrounded by open $]\emptyset[$ and in parts of these open, there exists distinct neigborhoods for $\emptyset_1$ and $(\emptyset_3)$. The space $S(\emptyset)$ is therefore Hausdorff separated. Units $(\emptyset)$ formed with parts thus constitute a topology $(T_\emptyset)$ of discrete space. Indeed, it also contains the discrete topology $(\emptyset^\emptyset, (\emptyset))$, which is the coarse one and is of much less mathematical interest.

Lemma 3. The magma of empty hyperset is endowed with self-similar ratios. The Von Neumann notation associated with the axiom of availability, applying on $(\emptyset)$, provides existence of sets $(N^\emptyset)$ and $Q^\emptyset$ equipotent to the natural and the rational numbers, Refs. [14, 15]. Sets Q and N can thus be used for the purpose of a proof. Consider a Cartesian product $E_n \times E_n$ of a section of $(Q^\emptyset)$ of n integers. The amplitude of the available intervals range from 0 to n, with two particular cases: interval $[0, 1]$ and any of the minimal intervals $[1/(n-1),\ 1/n]$. Consider now the open section $]0, 1[$: it is an empty interval, noted $\emptyset_1$. Similarly, note $\emptyset_{\min} =]0, 1/n(n-1)[$. Since interval [0, $1/n(n-1)]$ is contained in $[0, 1]$, it follows that $\emptyset_{\min} \subset \emptyset_1$. Since empty sets constitute the founding cells of the lattice $S(\emptyset)$ proved in Lemma 1, the lattice is tessellatted with cells (or balls) with homothetic-like ratios of at least $r = n(n-1)$.

Definition 1. Such a lattice of tessellation balls will be called a "tessellattice".

Lemma 4. The magma of empty hyperset is a fractal tessellattice.

Proof. (i) As follows from the above, one can write $(\emptyset) \cup (\emptyset) = (\emptyset,\ \emptyset) = (\emptyset)$.

(ii) It is straightforward that $(\emptyset) \cap (\emptyset) = (\emptyset)$.

(iii) Last, the magma $(\emptyset^\emptyset) = \{\emptyset,\ \complement\}$ represents the generator of the final structure, since $(\emptyset)$ acts as the "initiator polygon", and complementarity as the rule of construction. These three properties stand for the major features which characterise a fractal object [17].

Finally, the axiom of the existence of the empty set, added with the axiom of availability in turn provide existence to a lattice $S(\emptyset)$ that constitutes a discrete fractal Hausdorff space, and the proof is complete.

4. Existence and nature of spacetime

The existence of a Boolean lattice with fractal properties originating from nonwellfounded properties of the empty can be compared with the presence of a primary substrate with both discrete and continuous properties. Such a substrate acts in a role of a physical universe up to the function of conscious perception [5, 6].

In such a construction, spacetime is presented as an ordered sequence of mappings of closed 3D Poincaré sections of a topological 4-space provided by the lattice, and even the function of conscious perception is founded on the same properties.

Lemma 5. A lattice of empty sets ensure the existence of at least physical-like space.

Proof. Let $\emptyset$ denote the empty set as a case of the whole structure, and $\{\emptyset\}$ denotes some of its parts. The set of parts of $\emptyset$ contains parts equipotent to sets of integers, of rational and of real numbers, and owns the power of continuum [15, 18, 19]. The intersections of the inferring spaces (W^n), (W^m), ... having unequal dimensions lead to the appearance of spaces containing all their accumulation points and, thus, forming closed sets. Hence

$$\{(W^n) \cap (W^m)\}_{m>n} = (\Theta^n) \text{ is closed space.} \tag{14}$$

These spaces are discrete manifolds whose interior is endowed with the power of continuum. Consider a particular case (Θ^4) and the set of its parts $P(\Theta^4)$; any of intersections of subspaces $(E^d)_{d<4}$ provides a d-space in which the Jordan-Veblen theorem allows closed members to get the status of both observable objects and perceiving objects [14] . This is a condition for a space to be in some sort observable, that is physical-like. In any (Θ^4)-space, the ordered sequences of closed intersections $\{(E^d)_{d<4}\}$ with respect to mappings of members of $\{(E^d)_{d<4}\}_i$ into $\{(E^d)_{d<4}\}_j$ provides an orientation accounting for the physical arrow of time [16], in turn embedding an irreversible arrow of biological time [19].

Proposition 1. A manifold of potential physical universes is provided by the (Θ^4) category of closed spaces.

Thus, our spacetime is one of the mathematically optimum ones together with the alternative series of $\{(W^3) \cap (W^m)\}_{m>3}$. Higher spacetimes $(\Theta^n)_{n>3}$ could also exist.

4.1. Spacetime as a topologically discrete structure

The mappings of the Poincaré section S_i into the section S_{i+1} imposes the conservation of the topologies of the general structure of the mapped spaces. This makes it possible to characterise changes in the position of objects located inside these structures. A closed set should be mapped into an equivalent closed, an open into an equivalent open, and the places of points $(x, y, \dots)$ with respect to these reference structures $(A, B, \dots)$ are described by indicatrix functions $1A(x), 1B(x), \dots$

Two Poincaré sections, which are mapped, allow the assessment by using a natural metrics of topological spaces – the set-distance, Refs. [15, 16]. Let $\Delta(A, B, C, \dots)$ be the generalised set distance as the extended symmetric difference of a family of closed spaces:

$$\Delta(A_i)_{i \in N} = \complement_{\cup\{A_i\}} \bigcup_{i \neq j}(A_i \cap A_j) \quad (15)$$

The complementary of Δ, that is $\bigcup_{i \neq j}(A_i \cap A_j)$ in a closed space is closed. It is also closed even if it involves open components with nonequal dimensions. Thus, in this system $\mathcal{m}\langle\{A_i\}\rangle = \bigcup_{i \neq j}(A_i \cap A_j)$ is the instance, that is the state of objects in a timeless Poincaré section [14, 15]. Since distances Δ are the complementaries of objects, the system stands as a manifold of open and closed subparts. Mappings of these manifolds from one into another section, which preserve the topology, stand for a reference frame in which the "analysis situ" (the original name for topology) will allow one to characterise the eventual changes in the configuration of some components. If morphisms are observed, then it should be interpretable as a motion-like phenomenon when comparing the state of a section to the state of the mapped section.

It should be noted that the spaces referred above can exist upon acceptance of the existence of the empty set as a primary axiom [15, 16].

Bounias' Lemma [6]. The set-distance provides a set with the finer topology and the set-distance of nonidentical parts provides a set with an ultrafilter.

Proof. The set-distance Δ is founded on $\{\cap \cup \in\}$ and it suffices to define a topology since union and intersection of set-distances are distances, including $\Delta(A, A) = \emptyset$. The latter case must be excluded from a filter, which is nonempty. Then, since any filter and any topology is founded on $\{\cap \cup \in \notin \supset\}$, it is provided with Δ. Conversely, regarding a topology or a filter founded on any additional property $(\perp)$, this property is not necessarily provided to a Δ-filter. The topology and filter induced by Δ are thus respectively the finer topology and an ultrafilter.

The mappings of both distances and instances from one to another section can be described by a function called the "moment of junction" (MJ), which has the global structure of a momentum. Here is an example: the case of the homeo-morphic sequence of mappings of the general topology of the system; this provides a kind of reference frame, in which it will become possible to assess the changes in the situation of points and sets of points eventually present within these structures. The appropriate Bounias' lemma involving an indicator function is proved in paper [6].

So, the composition of the topological distances $\Delta(A, B, \dots) = \complement_{A \cup B} \dots (A \cap B \cap \dots)$ or the topological "instances" $\mathcal{m}\langle(A, B, C, \dots)\rangle = (A \cap B) \cup (A \cap C) \cup (B \cap C) \dots$ with a function *f,* which indicates the changes occurring in the situation of objects, acting over the populations of objects in the considered sets, leads to a momentum-like structure (MJ) and accounting for elements of the differential geometry of space.

The (MJ), mapping an instance (a 3D section of the embedding 4-space) to the next one, applies to both the open (the distances) and their complementaries, which are the closed (the reference objects) in the embedding spaces. Hence, points standing for physical objects may also be contained in both of the reference structures. Then, it appears that two kinds of mappings are composed with one another.

Bounias' Theorem [6]. A space-time-like sequence of Poincaré sections is a nonlinear con- volution of morphisms.

Proof demonstrates that the generalised convolution, which is a nonlinear and multidimensional form of the convolution product, exhibits a great similarity with a distribution of functions, namely in the Schwartz [20] sense

$$\langle f,\ \varphi\rangle = \sum \varphi(x) f(x), \tag{16}$$

or a convolution product

$$\int f(X-u)F(u)d(u) = (f * F)\,(X). \tag{17}$$

Thereby, the connection from the abstract universe of mathematical spaces and the physical universe of our observable ordinary space-time, i.e. the fundamental metrics, is provided by a convolution of morphisms, which supports the conjecture of the relation [21, 6]

$$\mathfrak{D}4 = \int \left(\int_{dS_0}^{dS_{\max}} (d\vec{x} \cdot d\vec{y} \cdot d\vec{z}) * d\Psi(x) \right) \tag{18}$$

where dS is the element of space-time and $d\Psi(x)$ is the function accounting for the extension of 3D coordinates to the 4th dimension through convolution ($*$) with the volume of space. In such a way, spaces of topologically closed parts account for the interaction and perception and hence they meet the properties of physical spaces.

5. Particles in a lattice universe

Let space be represented by the lattice $F(\mathrm{U}) = \{\cup\,(\sum_n W^n)\} \cup \varpi$ where ϖ is the set with neither members nor parts, i.e. the "nothingness singleton". The ϖ has neither members nor parts and it is contained in none of existing sets: otherwise it would be the complementary of Borel sets and therefore it would include parts of itself. This provides the set of possible structures with a lower boundary [6].

Thus, the lattice $F(\mathrm{U}) = \{\cup\,(\sum_n W^n)\} \cup \varpi$ supports both relativistic space and quantic void because (i) the concept of distance and the concept of time have been defined on it, and (ii) this space holds for a quantum void since on one hand, it provides a discrete topology, with quantum scales, and on the other hand it contains no "solid" objects that would be associated with physical matter.

The further consideration [6] of an indicator function involves the mapping of a frame of reference into its image frame of reference in the next section of spacetime. Without such a continuity there would be no possibility of assessing the motion of any object in the perceived universe (and this is exactly a case of "analysis situs" in the original meaning used by Poincaré). Continuity in the perception of a spacetime is provided iff the frames of references are conserved through homeomorphic mappings. This means that there is no need for exact replication: just topological structures should be conserved. Therefore, the implementation of varieties is allowed even in a space of different dimensions. This supports the following:

Proposition 2. The sequence of mappings of one into another structure of reference (e.g. elementary cells) represents an oscillation of any cell volume along the arrow of physical time.

There may be a threshold that prevents the preservation of homeomorphisms: let the cell transformation involve some repeated internal similarity (Fig. 2 shows a simplified example). Then, if N similar figures with similarity ratios $1/r$ are obtained, the Bouligand exponent (e) is given by the relation (8), $N(1/r)^{\mathrm{e}} = 1$, and the image cell gets a dimensional change from d to $d' = \ln(N)/\ln(r) = \mathrm{e} > 1$.

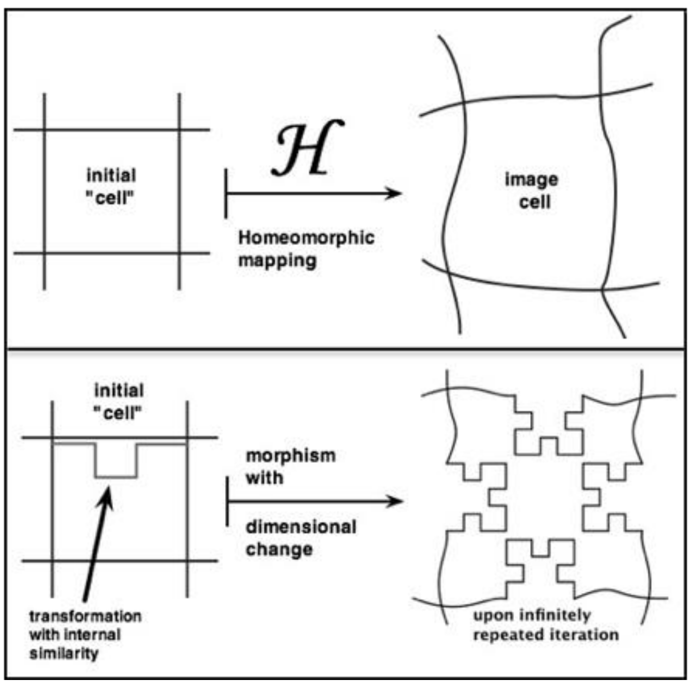

Figure 2. The continuity of homeomorphic mappings of structures is broken if once a deformation involves an iterated transformation with internal self-similarity, which involves a change in the dimension of the mapped structure. The first two or three steps of the iteration are sketched with basically the new figure jumping from (D) to approximately $(\mathrm{D} + 1.45)$. The mediator of transformations is provided in all cases by empty set units.

Then, probably, the homeomorphic part of the image cell is no longer an continued figure, and the transformed cell no longer possesses the property of a reference cell. This transformation stands for the formation of a "particle" also called "particled cell" or more appropriately "particled ball", since it is a kind of topological ball $B[\emptyset, r(\emptyset)]$. Thus we can claim the

Statement: A particled ball is represented by a non-homeomorphic transformation in the continuous deformation of elementary cells of space.

5.1. Quanta of fractality

To understand the interactions of particled balls with the degenerate space-lattice and further with other particled balls, it is necessary to demonstrate some mathematical preliminary operations.

A minimum fractal structure is provided by a self-similar figure whose combination rule includes a initiator and generator for which the similarity dimension exponent is higher than unity.

(i) **Initiator.** Due to self-similarity of $\geq \emptyset$, one considers the complementary of itself in itself and the one gains: $\emptyset \longmapsto \{(\emptyset), \emptyset\}$. That is, one ball gives two identical balls. This is continued into a sequence of $\{\frac{1}{2}, \frac{1}{4}, \dots, \frac{1}{2n}\}$ numbers at the n^{th} iteration and the series $(I) = \sum_{i=1\to\infty}\{1/2^i\}$ stands for the initiator providing the needed iteration process. The terms of (I) are indexed on the set of natural numbers, and thus supply an infinitely countable number of members.

Bear in mind, 2^n also denotes the number of parts from a set of n members.

(ii) **Generator**. Let an initial figure (A) be subdivided into r subfigures at the first iteration and hence the similarity ratio is $\varrho = 1/r$. Let $N = (r + a)$ be the number of subfigures constructed on the original one. Then one has $\mathrm{e} = -\mathrm{Ln}(r + a)/\mathrm{Ln}N$ and in this expression the value of e is bounded by unity if r is extended to infinity. For any finite r (which is presumably the case in the physical world), the exponent "e" is greater than unity. Then

$$\{\min(\mathrm{e})\,|\mathrm{e} > 1\} = \mathrm{Ln}\,(\max(r) + 1)\;/\;\mathrm{Ln}(\max\,(r)), \tag{19}$$

which completes the description of a quantum of fractality.

5.2. The fractal decomposition principle

Consider a fractal system Γ constructed as $\Gamma = \{(\emptyset), (r + a)\}$. More complex systems simply require combining several different subfigures, to which the following reasoning can be extended. At the n^{th} iteration, the number of additional subfigures is $N_n = (r + a)^n$ and the similarity ratio is $\varrho_n = 1/r^n$. At the i^{th} iteration, subvolume (v_i) is created and in the simplest case $v_i =$

$v_{i-1} \cdot (1/r)^3$. Since the number $N_i = (r+a)^i$ of such subvolumes is created at the i^{th} iteration, the total volume covered by the subvolumes formed by the fractal iterations to infinity is the sum of the series

$$v_f = \sum_{i=1\to\infty}\left\{(r+a)^i + v_{i-1}(1/r)^3\right\} \tag{20}$$

that can further be developed into

$$v_f = \sum_{i=1\to\infty}\left\{\left[\prod_{i=1\to n}\right](r+a)^{i-1}(1/r)^3\right\} \tag{21}$$

This leads to the following

Definition 2. Fractal decomposition consists in the distribution of members of a set of fractal subfigures

$$\Gamma \supset \left\{\sum_{i=1\to\infty}\{(r+a)^i \cdot v_{i-1} \cdot (1/r)^3\}\right\} \tag{22}$$

constructed on one figure among a number of connected figures $(\mathrm{C}_1, \mathrm{C}_2, \dots, \mathrm{C}_k)$ similar to the initial figure (A). If k reaches infinity, then all subfigures of (A) are distributed and (A) is no longer a fractal.

That is, a ball with its set of fractals can distribute these fractals all around to the nearest balls, such that the ball will lose its fractal dimension though preserving the volume (Fig. 3). Such a picture can arise during the motion of the particled ball, which squeezes between surrounding degenerate balls of the tessellattice and experiences a kind of friction. Fractal decomposition leads to the distribution of coefficients $f(\mathrm{e}_k)$, whose most ordered form is a sequence of decreasing values:

$$f(\mathrm{e}_k) = \{(\mathrm{e}_1)_{(i\,\in\,]k,1])}\}. \tag{23}$$

From the relation (23), it follows that the remaining of fractality decreases from the kernel (i.e. the zone adjacent to the original particled deformation) to the edge of the cloud of scattered fractals. At the edge, it can be conjectured that, depending on the local resistance of the tessellattice, the last decomposition (denoted as the n^{th} iteration) can result in $(\mathrm{e}_n) = 1$. Thus, while central fractals exhibit decreasing higher boundaries, edge fractals are bounded by a rupture of the remaining fractality.

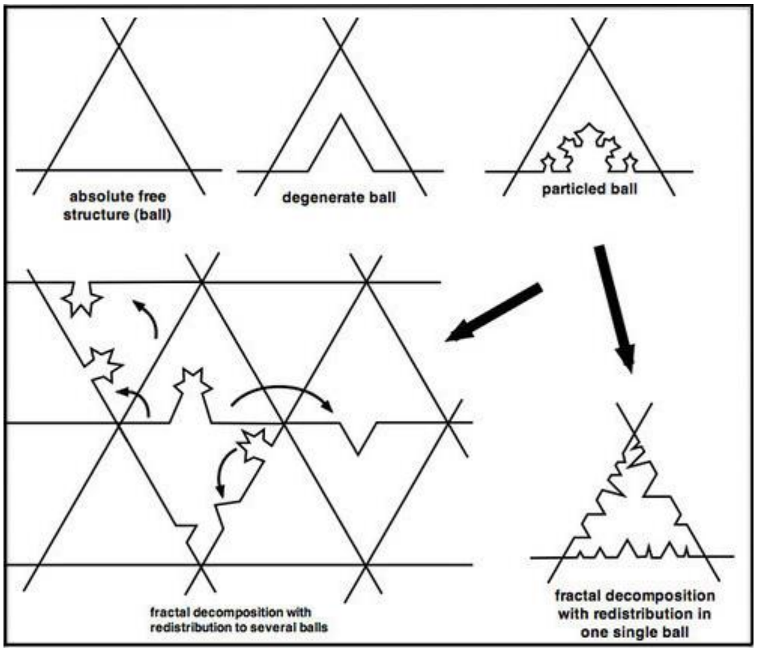

Figure 3. Scheme of the possible distribution of volumetric fractals from the particled ball.

5.3. Particle's fractals as fragments of mass

A particled ball, as described above, provides a formalism that describes elementary particles. In this respect, mass is represented by a fractal reduction in the volume of the ball, while a simple reduction in volume, as in degenerate cells, which is not obey any law, is not enough to provide mass. Accordingly, if v_{o} is the volume of an absolutely free cell, then the re- duction in volume as a result of fractal concavity will be as follows: $\mathcal{V}^{\text{particle}} = v_{\text{o}} - v_f$, that is, according to the relation (21):

$$\mathcal{V}^{\text{particle}} = v_{\text{o}} \cdot \left(1 - \sum_{i=1\to\infty}\{[\prod_{i=1\to n}]\ (r+a)^{i-1}/\,(r)^{3i}\}\right) \qquad (24)$$

that is, since (r + a) = (r)$^{\text{e}}$, we have instead of (24)

$$\mathcal{V}^{\text{particle}} = v_{\text{o}} \cdot \left(1 - \sum_{(v)} \left(\sum_{i=1\to\infty} \{[\prod_{i=1\to n}]\,(r_v)^{\text{e}_v(i-1)}/\,(r)^{3i}\}\right)_v \right) \qquad (25)$$

where (v) denotes several possible fractal concavities affecting the particled ball.

The relationship (25) relates the volume of particled balls to the fractal dimensional change (e), which can be expressed as the following:

Proposition 3. The mass m of a particled ball A is a function of the fractal-related decrease of the volume of the ball:

$$m \propto \left(\mathcal{V}^{\text{degen.}} / \mathcal{V}^{\text{particle}}\right) \cdot (\mathrm{e}_v - 1)_{\mathrm{e}_v > 1} > 1 \quad (26)$$

where $\mathcal{V}^{\text{degen.}}$ is the volume of a degenerate topological ball, $\mathcal{V}^{\text{particle}}$ is the volume of the particled ball, (e) is the Bouligand exponent and $(\mathrm{e} - 1)$ is the gain in dimensionality given by the fractal iteration as ascribed to the volumetric changes of the ball. If we multiply expression (25) by the dimensional factor C, we will have the physical definition of the mass of the particle.

Just a volume decrease is not sufficient for providing a ball with mass, since a dimensional increase is a necessary condition. A ball contracted in the described way becomes a lepton particle, namely: an electron, muon, tau (and also their antipodes). So, the emergence of a local fractal deformation in the degenerate tessellattice means the appearance of matter.

6. Discussion and Conclusion

Thus, the conducted studies of the topology of physical space show that it consists of discrete cells endowed with quantum-determined relative scales and the interior of which is potentially provided with the power of continuum. This property bridges the gap between the still discrete nature of the microscopic world and the apparent continuity of the macroscopic universe.

The moments of junction map Poincaré’s timeless section representing the state of involved spaces into another state. The moments of junction represent the interval between two successive states (each timeless) of a universe. Let E_i be a Poincaré section like S_i defined above: if it is an identity mapping, $MJ = Id(S)$, then there is no time interval from S_i to S_{i+1}. In all other cases, the MJ represents two important parameters: first, it accounts for a differential time interval, and then for a differential element of the geometry of the corresponding space. In this sense, it has neither “thickness” nor duration. There is no “distance” in the Hausdorff sense between S_i and S_{i+1}, just a change in the topological situation. Since the step from S_i to S_{i+1} is a discrete one, it follows that (i) the corresponding space has discrete, i.e. peculiar quantum properties, and (ii) these discrete properties are valid independent of scales because they are based on a set difference that is neither scale nor size dependent phenomenon. It should be noted that these properties meet some requirements for space, time and matter.

The moment of connection formalizes the topological characteristics of what is called motion in the physical universe, that is, what is considered necessary for understanding physics. Whereas an identical mapping means no motion, i.e., zero time span, a nonempty moment of junction means the minimum of any time interval. In our understanding, there is no such “point”: only instances that in themselves do not reflect timely features.

The fractal kernel stands for a "particle" and the reduction of its volume (together with an increase of its area) is compensated by morphic changes of a finite number of surrounding cells.

Note the formalism of quantum mechanics was developed at the atom scale, 10^{-10} m; the space surrounding the particle is still considered to be empty, it is a vague vacuum. However, we investigate physical space practically in the vicinity of a point. Are there physical constraints that indicate the existence of drastic topological fluctuations at the Planck scale, i.e. $\sim 10^{-35}$ m? A mathematical space can give raise to several topologies, which range from coarser to finer forms, in relation to order [22]. A smoothing of topologies at low scale would be needed. These apparent contradictions disappear with the properties of the empty hyperset, which provides discrete features at all scales but also possesses the power of continuum, i.e. physical 'continuity' within each fundamental cell. Note that continuity in the mathematical sense does not require smoothing. The set-distance is a scale-independent measure that is capable of satisfying the necessary requirements, eliminating the scale-related problem. No contradiction seems to lurk in these approaches.

This is in direct agreement with the recent study of Haug [23], who shows that in a system with a huge number of particles, such as the Earth, the effective Compton wavelength reaches a value of $\sim 10^{-68}$ m, which is much smaller than the fundamental Planck length $\ell_{\mathrm{P}} = \sqrt{\hbar G / c^3} = 1.616 \times 10^{-35}$ m. Nevertheless, this effective length does work and this means that the Planck length is not minimal, and that space allows fragmentation to practically infinitesimally small values via fractals, as illustrates Fig. 3 (although the size of a cell of the tessellattice remains at the level of the Planck length).

So, the analysis of the topology of physical space discussed in this paper shows that physical space, i.e. our universe, exists in the form of a tessellattice, i.e. a mathematical lattice tightly packed with primary topological balls. These balls play the role of cells of the tessellattice. Cells with the size equal to ℓ_{P} are rather in a degenerate state, and a particle is created from a cell almost instantly in a rapid fractal transformation process (Fig. 2).

The creation of a particle means a complete volumetric fractal transformation of the cell, and the particle pattern is given by expressions (24) and (25). The appearance of something (i.e. a matter) in the emptiness (the degenerate tessellattice) is characterised by the particle mass, which is defined in the relation (26). The transition from the mathematical definition of the particle mass (26) to the physical value must include the dimensionality factor.

The motion of the particle occurs with the decay of its mass – a cloud of fractals, i.e. mass fragments, accompany the particle. These mass fragments were named 'inertons' by the author (see, e.g. Ref. [9]), and it was shown that the inerton cloud emitted by the moving particle is reflected by the elastic tessellattice back to the particle. So, the particle periodically emits its inerton cloud along each odd section $\lambda/2$ of its path, and then absorbs inertons back along each even section $\lambda/2$. Then the length λ should be called the period of spatial oscillations of the particle, and this magnitude itself is nothing but the particle de Broglie wavelength. The two relationships proposed by de Broglie

[24], were also derived by the author based on consideration of the motion of a particle in the tessellattice, i.e. from first principles:

$$\lambda = h/mv, \qquad E = h\nu \tag{27}$$

where $\nu = v/\lambda$. These two relationships allow one to easily derive the Schrödinger equation (see, e.g. Ref. [9]):

$$\Delta\psi(x,t) + i\frac{2m}{\hbar}\frac{\partial}{\partial t}\psi(x,t) = 0. \tag{28}$$

Besides, the relationships (27) satisfy also a conventional wave equation

$$\Delta\psi(\vec{r},t) - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\psi(\vec{r},t) = 0, \tag{29}$$

which is the only one that correctly describes the spectra of all atoms. Namely, Shpenkov and Kreidik [25–32] in their very impressive and important works showed that the Schrödinger equation that includes the Coulomb potential offers an artificial not natural spectrum, although physicists for decades have emotionally explained the success of quantum mechanics using the hydrogen atom as an example. They [25-32] really demonstrated that contrary to the Schrödinger equation (28), the wave equation (29) that propagates in the real medium perfectly describes all experimental spectra of different atoms starting from the hydrogen one. Thereby, physical space presented in the form of the tessellattice, which is a substrate, in fact allows the propagation of real waves and hence in such a case the description of microphysical phenomena using the wave equation looks quite natural.

In general, we see that the tessellattice plays the role of neutrality, that is, a state where no physical parameter dominates over the other, such as positive and negative charge, mass and tension, cell inflation and contraction, left and right vortex, etc.

Further research [9] revealed how far the particle's inerton cloud spreads from the particle and the appropriate amplitude of the inerton cloud is equal to $\Lambda = \lambda c/v$ where c is the speed of light, which is also the speed of sound of the tessellattice. Thus, the system {particle + its inerton cloud}, i.e. the kern ball with its disseminated volumetric fractals, is mapped into the quantum mechanical formalism as the "mysterious" ψ-wave function of the particle or more naturally is included in the wave equation. Thus, realistically the wavefunction $\psi(r)$ represents the distribution of mass $m(r)$ of the system {particle + its inerton cloud} normalised to the initial rest mass m_0 of the particle, i.e. $\psi(r) = m(r)/m_0$.

Of course, some inertons can be released from the particle's inerton cloud and migrate individually, hopping from cell to cell in the tessellattice. Hence, inertons as carriers of mass are also carriers of the field of inertia, which can be considered as a new physical field discovered recently. It is likely that in the near future an inerton communication may compete with wireless electromagnetic communication [33].

In a similar way, in addition to the considered volumetric fractals, we can also examine surface fractals of primary topological balls in the tessellattice. This allows us to derive the concept of electric charge as a spherical surface covered with surface fractals, which are spikes [34, 7, 9]. Finally, one can arrive at Maxwell's equations [9, 35], which represent not only an electric charge, but also a magnetic monopole, since only a monopole can be a source of a magnetic field.

Besides, in the tessellattice, fractals can not only contract the volume of cells, fractals are able also to inflate cells increasing the ball's volume, which means the creation of a family of quarks [7 – 9].

Thus, the described approach to the study of topology of physical space paves the way for a deeper investigation of microcosm than the quantum mechanical formalism presented. Basically, this submicroscopic approach is resting on the theory of real space, discussed in this paper, and allows a radical revision of existing abstract concepts that prevail in theoretical models of particle physics. The submicroscopic concept enables also to thoroughly study the origin of gravity and understand such unusual phenomena as dark matter and dark energy in detail [9, 36].

Finally, let us look into the distant past. Vedic heritage and epics are very interesting and instructive. That culture started in the country of Aratta 8000–5000 years ago [37] (known as Cucuteni-Trypillia archaeological culture that existed in the lower reaches of the Dnipro and Danube rivers covering modern countries of Ukraine, Moldova and Romania). Around 3,000 years ago, their descendants migrated to Punjab where the Vedic texts were written down by brahmins. Some of the records are directly related to the issues discussed in this paper.

Roy [38] reading *The Rigveda* and other ancient Vedic books as a physicist saw in the texts the coded knowledge about the structure of space, cosmology and elementary particles. In particular, he found that real space was called 'loka': the loka has a web structure, it consists of indivisible cells; cells are characterised by their interface. Then he derived the notion of the electric charge from a few verses of *The Rigveda*: "The electric charge is kept and plays on the surface of the particle", which exactly coincides with the structure of the electric charge proposed in [7, 9, 34, 35] as discussed above.

Let us read *The Bhagavad-Gita* together with Bhaktivedanta Swami Prabhupada [39], the known religious scholar. Chapter 2, which is a review of *The Bhagavad-Gita*, contains the information on the existence of the first cause of matter in the form of an indivisible thing that is usually called "soul". However, Swami Prabhupada [39] noted that soul should also be understood as a **subtle particle**. Then we can read in Chapter 2: "In spite of the material body being subject to destruction, the subtle particle is eternal" (*BhG*.: 2.18); "It never takes birth and never dies at any time nor does it come into being again when the material body is created. It is birthless, eternal, imperishable and timeless and is inviolable when the body is destroyed" (*BhG*.: 2.20); "After some time it is disenthralled by entire annihilation of the material body. Yet it endures the destruction of the material world" (*BhG*.: 2.22); "It is not fissionable, not burning out, not soluble, and not drying up" (*BhG*.: 2.23); "Since it is not visible, its entity does not change, its properties remain unchangeable" (*BhG*.: 2.24). Then in Chapter 8 we can read in addition: "Yet there is another nature, which is eternal and is transcendental to this manifested and unmanifested matter. It is supreme and is never annihilated. When all in this world is annihilated, that part remains as it is" (*BhG*.: 8.20).

Thereby, *The Bhagavad-Gita* appears to reveal, in particular, information about the building block of real space that exists in the form of a web net. Such a block, the subtle particle cannot annihilate; it is eternal and cannot be destroyed. This knowledge on the structural block of space is an additional confirmation of the correctness of Roy's [38] deciphering: real physical space has the structure of a web net (and each cell of the net is occupied by a "subtle particle"). Vedic literature also informs us that the subtle particle (or superparticle, or primary topological ball) is primordial and incomprehensible.

Nowadays researchers mostly follow the path of a logical-analytical, rational worldview, which can be associated with "manual control" and almost complete disregard for learning on the part of Space. In modern times, as we see in social life, such manual control everywhere brings only disorder and destruction. However, not only physics, but also all mathematics, including topology and fractal geometry, begin with the laws of real Space, and therefore it is Space that is capable of giving science the correct guidelines regarding further directions of research.

During the times of Vedic culture (8,000–5,000 years ago in the country of Aratta), people's worldview rested on a figurative-intuitive perception of the world, which subconsciously brought culture into the information field [40] (it was manifested through the all-powerful, all-seeing god of the Slavic, Indian and Iranian Vedas). Therefore, that society lived without wars, conflicts and strifes; there were not even mournful songs. Such a perception of the world automatically introduced a person into natural harmonic behaviour, as if the person was in the "autopilot mode", and then the laws of Space subconsciously guided the person through life. For this, one needs to make an effort to connect to this "autopilot" through understanding and psychophysical practice, that is, to connect to the natural harmony of Space and the presence of oneself in Space as well.

So scientists who largely follow a logical-analytical, rational worldview must also realise the presence of a natural figurative-intuitive worldview, which is also capable of leading to significant achievements and therefore should never be neglected.

References

[1] D. S. Simon, *Tying Light in Knots. Applying Topology to Optics*, (IOP Concise Physics, 2018).

[2] J. Davighi, "Topological effects in particle physics phenomenology" (2020). https://doi.org/10.17863/CAM.47560

[3] T. Maudlin, "Time and the geometry of the universe", Geometry and Topology, Academia (2025) https://www.academia.edu/8412496/Time_and_Geometry

[4] C. Mortensen, G. Nerlich, "Physical topology", Journal of Philosophical Logic, **7**, 209-223 (1978). https://doi.org/10.1007/BF00245928

[5] M. Bounias, and V. Krasnoholovets, "Scanning the structure of ill-known spaces: Part 1. Founding principles about mathematical constitution of space", The Kybernetes: The Int. J. Systems and Cybernetics, **32**(7/8), 945-975 (2003); also arXiv:physics/0211096.

[6] M. Bounias, and V. Krasnoholovets, "Scanning the structure of ill-known spaces: Part 2. Principles of construction of physical space", Ibidem, **32**(7/8), 976-1004 (2003); also arXiv:physics/0212004.

[7] M. Bounias, and V. Krasnoholovets, "Scanning the structure of ill-known spaces: Part 3. Distribution of topological structures at elementary and cosmic scales", Ibidem, **32**(7/8), 1005-1020 (2003); also arXiv:physics/0301049.

[8] M. Bounias, and V. Krasnoholovets, "The universe from nothing: A mathematical lattice of empty sets", (Daniel M. Dubois, Ed.): CASYS'2003, International Journals of Anticipatory Computing Systems, **16**, 3-24 (2004), also arXiv:physics/0309102.

[9] V. Krasnoholovets, *Structure of Space and the Submicroscopic Deterministic Concept of Physics*, (Apple Academic Press (CRC Press), Toronto • New York • New Delhi, 2017).

[10] A. Litinas, S. Geivanidis, A. Faliakis, Y. Courouclis, Z. Samaras, A. Keder, V. Krasnoholovets, I. Gandzha, Y. Zabulonov, O. Puhach, and M. Dmytriyuk, "Biodiesel production from a high FFA feedstock with a chemical multifunctional process intensifier", Biofuel Research Journals, **7**(2), 1143-1177 (2020).

[11] V. Krasnoholovets, and V. Fedorivsky, "Transmission of wellness information signals using an inerton field channel", *Eurean Journal of Applied Physics*, **2**(6), 1-8 (2020).

[12] C. Tricot, *Courbes et Dimension Fractale*, (Springer-Verlag, Berlin, Heidelberg, 1999).

[13] M. Bounias, and A. Bonaly, "On mathematical links between physical existence, observability and information: towards a "theorem of something", Journal of Ultra Scientist of Physical Sciences **6**(2), 251-259 (1994).

[14] M. Bounias, and A. Bonaly, "On metrics and scaling: physical coordinates in topological spaces", Indian Journal of Theoretical Physics, **44**(4), 303-321 (1996).

[15] M. Bounias, and A. Bonaly, "The topology of perceptive functions as a corollary of the theorem of existence in closed spaces", BioSystems, **42**, 191-205 (1997).

[16] M. Bounias, and A. Bonaly, "Some theorems on the empty set as necessary and sufficient for the primary topological axioms of physical existence", Physics Essays, **10**(4), 633 - 643 (1997).

[17] G. James, and R. C. James, *Mathematics Dictionary*, (Van Nostrand Reinhold, New York, 1992), p267-268.

[18] M. Bounias, "The theory of something: a theorem supporting the conditions for existence of a physical universe, from the empty set to the biological self", in (Ed.: Daniel M. Dubois): CASYS'99 Int. Math. Conf., International Journal of Computational Anticipatory Systems, **5**, 11-24 (2000).

[19] M. Bounias, "A theorem proving the irreversibility of the biological arrow of time, based on fixed points in the brain as a compact, delta-complete topological space", in (Ed.: Daniel M. Dubois) CASYS'99 Int. Math. Conf., AIP Conference Proceedings, **517**, 233-243 (2000).

[20] L. Schwartz, *Mathematics for the Physical Sciences*, (Addison-Wesley Publishing Company, Reading, MA, 1966), Ch. III.

[21] A. Bonaly and M. Bounias, "The trace of time in Poincare sections of topological spaces", Physics Essays, **8**(2), 236-244 (1995).

[22] N. Bourbaki, *Topologie Gendrale*, (Masson, Paris, 1990), Chs. 1-4, p376.

[23] E. G. Haug, "Planck scale fluid mechanics: Measuring the Planck length from fluid mechanics independent of G", Open Journal of Fluid Dynamics **13**(5), 250–261 (2023).

[24] L. de Broglie, "Recherches sur la théorie des Quanta", Annales de Physique, 10^{e} série, t. III, 22-128, (1925); English translation by A. F. Kracklauer: *On the Theory of Quanta*, Lulu.com, (Morrisville, NC, 2007).

[25] L. Kreidik, and G. Shpenkov, *Atomic Structure of Matter-Space*, (Geo. S., Bydgoszcz, 2001), Ch. 3, p119-185.

[26] G. P. Shpenkov, and L. G. Kreidik, "Microwave background radiation of hydrogen atoms", Revista Ciências Exatas e Naturais**,** **4**(1), 9-18 (2002).

[27] L. G. Kreidik, and G. P. Shpenkov, "Important results of analyzing foundations of quantum mechanics", Galilean Electrodynamics & QED-EAST, **13**(2), 23-30 (2002).

[28] G. P. Shpenkov, and L. G. Kreidik, "Dynamic model of elementary particles and fundamental interactions", Galilean Electrodynamics, Special Issue – GED East, **15**(2), 23-29 (2004).

[29] G. P. Shpenkov, "The nodal structure of standing spherical waves and the periodic law: What is in common between them?", Physics Essays, **18**(2), 196-206 (2005).

[30] G. P. Shpenkov, and L. G. Kreidik, "Schrödinger's error in principle", Galilean Electrodynamics **16**(3), 51-56 (2005).

[31] G. P. Shpenkov, "An elucidation of the nature of the periodic law", in: *The Mathematics of the Periodic Table,* Eds: D. H. Rouvray, and R. B. King, (Nova Science Publishers, New York, 2006), Ch. 7, p119-160.

[32] L. Kreidik, and G. Shpenkov, "An analysis of the basic concepts of quantum mechanics and new (dialectical) solutions for the fields of a string and H-atom", in *Old Problems and New Horizons in World Physics,* Eds.: V. Krasnoholovets, V. Christianto, and F. Smarandache, (Nova Science Publishers, New York, 2020), Ch. 2, pp. 5–89.

[33] V. Krasnoholovets, "Inerton communication: Future of wireless", European Journal of Applied Physics, **4**(4), 28-32 (2022).

[34] V. Krasnoholovets, "On the nature of the electric charge", Hadronic Journal Supplement, **18**(4), 425-456 (2003); also arXiv:physics/0501132.

[35] V. Krasnoholovets, "Magnetic monopole as the shadow side of the electric charge", Journal of Physics: Conference Series, **1251**:012028 (2019). Conference Proceeding SEARCH FOR FUNDAMENTAL THEORY: The IX International Symposium Honoring French Mathematical Physicist Jean-Pierre Vigier (12-14 August 2018, Liege), Eds.: R. L. Amoroso, and D. Dubois; also arXiv:2106.10225.

[36] V. Krasnoholovets, "Derivation of gravity from first submicroscopic principles", in *The Origin of Gravity from First Principles*, Ed.: V. Krasnoholovets, (Nova Science Publishers, New York, 2021), p281-332.

[37] Yu. Shilov, *Ancient History of Aratta-Ukraine (20,000 BCE – 1,000 CE)*, (Amazon, 2015). https://www.amazon.com/Ancient-History-Aratta-Ukraine-000-BCE/dp/1505241626

[38] R. R. M. Roy, *Vedic Physics*, *Scientific Origin of Hinduism* (Golden Egg Publishing, Toronto, 1999).

[39] Shri Shrimad A. C. Bhaktivedanta Swami Prabhupada, *Beyond Time and Space* (Muscovite version of *Easy Journey to Other Planets*) (Almqist & Wiksell, Uppsala, 1976), p13-19; *Bhagavad-gita as it is* (Bhaktivedanta Book Trust, Moscow, Leningrad, Calcutta, Bombay, New Delhi, 1984), Ch. 2, Texts 17–30.

[40] V. Krasnoholovets, "Information field and its carriers in biological systems". NeuroQuantology, **20**(4), 179-201 (2022).