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Chapter 3

THE WAVE BEHAVIOR OF QUANTUM SYSTEMS AND THE SUBMICROSOCPIC CONCEPT OF THE MICROWORLD

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ABSTRACT

In some recent papers (G. 't Hooft and others) it has been argued that quantum mechanics can arise from classical cellular automata. Nonetheless, G. Shpenkov has exibited that the classical wave equation makes it possible to derive a periodic table of elements, which is very close to Mendeleyev's one, and describe also other phenomena related to the structure of molecules. Hence the classical wave equation complements Schrödinger's equation, which means the appearance of a cellular automaton molecular model starting from classical wave equation. The other studies show that the microworld is constituted as a tessellation of primary topological balls. The tessellattice becomes the origin of a submicrospic mechanics in which a quantum system is subdivided to two subsystems: the particle and its inerton cloud, which appears due to the interaction of the moving particle with oncoming cells of the tessellattice. The particle and its inerton cloud periodically change the momentum and hence move like a wave. The new approach allows us to correlate the Klein-Gordon equation with the deformation coat that is formed in the tessellatice around the particle. The submicroscopic approach shows that the source of any type of wave movements including the Klein-Gordon, Schrödinger, and classical wave equations is hidden in the tessellattice and its basic exciations – inertons, carriers of mass and inert properies of matter.

Keywords: Schrödinger equation, Klein-Gordon equation, classical wave equation, Shpenkov's model, periodic table of elements, molecule, cellular automata, submicroscopic concept, tessellettice, inertons

INTRODUCTION

Elze [1] and Shpenkov [2] wrote about possible re-interpretation of quantum mechnics (QM) starting from classical automata principles. This is surely a fresh approach to QM, based on some authors including 't Hooft [3]. In a series of papers Shpenkov [2, 3-14] draws attantion to the fact that the spherical solution of Schrödinger's equation says nothing about the structure of molecules; the spherical solution and its comparison with experimental data are hardly discussed properly in textbooks, with an excuse that it is too complicated.

According to Shpenkov [2, 3-14], the classical wave equation is able to derive a periodic table of elements, – which is close to Mendeleyev's periodic table, – and also other phenomena related to the structure of molecules. Hence it seems Shpenkov's interpretation of classical wave equation can complement Schrödinger equation.

However, the Schrödinger equation is a quantum equation that desdcribes the motion of the appropriate particle-wave since all quantum objects manifest charcatiristics of both particles and waves. Considering Shpenkov's results, one can ask why do the particle's characteristics dissapier and what exactly is the subject of purely wave behaviour in a quantum system?

In order to answer these questions, we involve recent studies of Krasnoholovets (see, e.g. Ref. [15]), in which a submicroscopic concept has been developed. Namely, it has been shown how the motion of a canonical particle occurs in physical space constructed as a cellular structure named the tessellattice.

In this work we carry out studies of the Schrödinger equation and wave equation and show how they both are related to the tessellatice that forms their essence and fills with physical content.

SCHRÖDINGER EQUATION VS. CLASSICAL WAVE EQUATION (OF SOUND)

Shpenkov's work is based on [13]: (1) Dialectical philosophy and dialectical logic; (2) The postulate on the wave nature of all phenomena and objects in the universe. He uses the classical wave equation

$$\Delta\hat{\Psi} - \frac{1}{c^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0. \tag{1}$$

This equation is also known as the wave equation of sound or string vibration (see, e.g.Refs. [16, 17]).

Shpenkov shows that the classical wave equation is able to:

a. Derive a periodic table of elements (slightly different from but close to the Mendeleyev's periodic law) based on spherical solution of his standing wave equation [10];
b. Give a dynamical model of elementary particles [6];
c. Derive binding energy of deuterium, tritium, helium and carbon [9];
d. Derive the atom background radiation of hydrogen which corresponds to the observed COBE/CMBR (Cosmic Microwave Background Radiation) [4];
e. Derive the shell-nodal model of atoms and molecules [7];
f. Explain anisotropy of graphene [13];
g. Describe the shell-nodal picture of carbon and grapheme [12];
h. Describe electron "orbitals";
i. Describe electron "spin";
j. Derive neutron magnetic moment;
k. Derive proton magnetic moment;
l. And other things [11].

Therefore, it seems that Shpenkov's wave model of particles and molecules may be a promising alternative to complement the standard quantum/wave mechanics.

Besides, Shpenkov points out to some serious weaknesses associated with (spherical solution of) Schrödinger's equation of a quantum system, which is in the potential field of one or another nature:

i. Its spherical solution is rarely discussed completely, though the complete spherical solution of Schrödinger's wave equation does not agree with any experiment.
ii. The Schrödinger equation is able only to arrive at hydrogen energy levels, and it has to be modified and simplified for other atoms. For example, physicists are forced to use an approximate approach called *Density Functional Theory* (DFT) in order to deal with an N-body system [18].
iii. The introduction of variable wave number k into the Schrödinger equation, depending on electron coordinates, and the omission of the azimuth part of the wave function, were erroneous [19]. Schrödinger's variable wave number should be questioned because the potential function cannot influence the wave speed or consequently the wave number.
iv. Introduction of the potential function V into the wave equation, which results in dependence of the wave number k on the Coulomb potential, *generates divergences* that do not have a physical justification. They are eliminated in an artificial way [5, p. 27].
v. Modern physics erroneously interprets the meaning of polar-azimuthal functions in Schrödinger's equation, ascribing these functions to atomic "*electron orbitals*" [14, p. 5].
vi. Schrödinger arrived at a correct result of hydrogen energy levels using only a radial solution of his wave equation, with one major assumption: the two quantum numbers found in the solution of his wave equation were assumed to be the same with Bohr's quantum number [8].
vii. Quantum mechanics solutions, in their modern form, contradict reality because on the basis of these solutions, the existence of crystal substances-spaces is not possible [5, p. 26].
viii. Schrödinger's approach yields abstract phenomenological constructions, which do not reflect the real picture of the micro-world [8].
ix. Schrödinger himself in his 1926 paper apparently wanted to interpret his wave equation in terms of vibration of string [20, 21]. This is why he did not accept Born's statistical interpretation of his wave equation until he died.

In the initial variant, the Schrödinger equation had the following form [8]:

$$\Delta\Psi + \frac{2m}{\hbar^2}\left(W + \frac{e^2}{4\pi\varepsilon_0 r}\right)\Psi = 0 \tag{2}$$

The wave function satisfying the wave equation (2) is represented as

$$\Psi = R(r)\,\Theta(\theta)\,\Phi(\varphi)\,T(t) = \psi(r,\theta,\varphi)\,T(t) \tag{3}$$

where $\Psi(r,\theta,\varphi) = R(r)\,\Theta(\theta)\,\Phi(\varphi)$ is the complex amplitude of the wave function because

$$\Phi_m(\varphi) = C_m\,e^{\pm im\varphi} \tag{4}$$

For standard method of separation of variables to solve the spherical Schrödinger equation (see e.g. Refs. [22, 23]).

Equations for the funtions Φ, Θ and *T* are known in the theory of wave fields. Hence these equations presented nothing new. Only the *R* was new. Its solution turned out to be *divergent*. However, Schrödinger together with H. Weyl (1885-1955), contrary to the logic of and all experience of theoretical physics, artificially cut off the divergent power series of the radial function *R*(*r*) at a *κ*-th term. This allowed them to obtain the radial solutions, which, as a result of the cut off operation, actually were the fictitious solutions [8].

Furthermore, it can be shown that the time-independent Schrödinger equation [22]

$$\nabla\Psi + \frac{2m}{\hbar^2}(E - V)\Psi = 0 \tag{5}$$

can be written in the form of standard wave equation [8]

$$\nabla\Psi + k^2\Psi = 0 \tag{6}$$

where

$$k = \pm\sqrt{\frac{2m}{\hbar^2}(E - V)}\,. \tag{7}$$

Comparing equations (2), (5) and (6), we obtain [8]

$$k = \pm\sqrt{\frac{2m}{\hbar^2}\left(W + \frac{e^2}{4\pi\varepsilon_0 r}\right)}, \tag{8}$$

which means that the wave number k in Schrödinger's radial wave equation is a quantity that *varies continuously in the radial direction*. Is it possible to imagine a field where the wave number, and hence the frequency, change from one point to another in the space affected by the field? Of course, it is not possible. Such wave objects do not exist in Nature.

SHPENKOV'S INTERPRETATION OF CLASSICAL WAVE EQUATION

Shpenkov suggested starting the study of the wave equation in the form

$$\nabla\Psi + \frac{\omega^2}{c^2}\Psi = 0 \tag{9}$$

instread of the standard classical wave equation (1). In eq. (9) the wave number $k = \omega / c$ is invariable [5]. Here, ω denotes a fundamental carrying frequency of the wave field at the corresponding level of space, and c denotes the speed of light. In order to correct the faults of wave mechanics, it is necessary to write down the above wave equation that meets the conditions: (a) the wave number is constant, and (b) the azimuth factor must be taken into consideration along with radial $R(r)$ and polar factor of the wave-function [5].

In this case, the differential equation for the radial factor $R(r)$ is:

$$\rho\frac{d^2 R_l}{d^2\rho} + 2\rho\frac{dR_l}{d\rho} + \left[\rho - l(l+1)\right]R_l = 0 \tag{10}$$

where $\rho = kr$.

Shpenkov suggests a kind of a fundamental wavelength corresponding to expression (9) equal to

$$\lambda\!\!\!^{-}_e = c / \omega_e = 1.603886998 \times 10^{-10} \text{ m}, \quad \omega_e = 1.86916197 \cdot 10^{18}\,\text{s}^{-1} \tag{11}$$

because it is one-half of a mean value of the interatomic distance in a solid.

The detailed analysis to find the spherical solution of equation (9) is discussed in Shpenkov's other papers [10, 7]. Some consequences of the solution of Shpenkov's interpretation of the classical wave equation are [5]:

a. As masses of atoms are multiple of the neutron mass (or hydrogen atom mass), following Haüy's ideas makes it reasonable to suppose that any atom, like the elementary Haüy's molecule, is the neutron (H-atom) molecule.

b. Therefore, atoms should be considered as neutron (H-atom) quasi-spherical multiplicative molecules. The word 'multiplicative' means that strong bonds must couple particles constituted of these elementary molecules, which we call multiplicative bonds.
c. Potential polar-azimuthal nodes of spherical shells in stable atoms (nucleon molecules) contain by two coupled nucleons.
d. Polar potential-kinetic nodes (not filled with nucleons in the most abundant and stable atoms) are ordered along the z-axis of symmetry (in spherical coordinate system) of the atoms.
e. Exchange (interaction) between completed nodes inside (strong) and outside (electromagnetic) of the atoms is realised by exchange charges of with the fundamental frequency (11).
f. Principal azimuth nodes of the wave space of atoms are marked by ordinal numbers. These numbers coincide with the ordinal numbers of elements of Mendeleyev's periodic table. The quantity of neutrons, localized in one node, is equal to or less than two.
g. Arranging atoms with the same or similar structure of outer shells one under another, one arrives at the *periodic-nonperiodic law of spherical spaces* that constitutes periodic table, slightly differing from the conventional one of Mendeleyev.

There are of course other researchers who use the classical wave equation to study atoms and particles (see e.g. Mills [24] and Close [25]).

CORRESPONDENCE BETWEEN CLASSICAL WAVE EQUATION AND QUANTUM MECHANICS

A connection between classical and quantum mechanics has been studied at least by several researchers (see e.g. Refs. [26-28]). Ward and Volkmer [29] discussed a relation between the classical electromagnetic wave equation and Schrödinger equation. They derived the Schrödinger equation based on the electromagnetic wave equation and Einstein's special theory of relativity. They began with electromagnetic wave equation in one-dimensional case

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 . \tag{13}$$

This equation is satisfied by plane wave solution:

$$E(x,\ t) = E_0 e^{i(kx-\omega t)}, \tag{14}$$

Where $k = 2\pi / \lambda$ and $\omega = 2\pi\nu$ are the spatial and temporal frequencies, respectively. Substituting equation (14) into (13), then we obtain

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) E_0 e^{i(kx-\omega t)} = 0, \tag{15}$$

or

$$\left(k^2 - \frac{\omega^2}{c^2} \right) E_0 e^{i(kx-\omega t)} = 0, \tag{16}$$

which arrives us to a dispersion relationship for light in free space: $k = \omega / c$. This is similar to the wave number k in eq. (8).

Then, recall from Einstein and Compton that the energy of a photon is $\varepsilon = h\nu = \hbar\omega$ and the momentum of a photon is $p = h / \lambda = \hbar k$, which allows us to rewrite eq. (14) using these relations:

$$E(x,\ t) = E_0\, e^{\frac{i}{\hbar}(px-\varepsilon t)}. \tag{17}$$

Substituting expression (17) into eq. (13) we find

$$-\frac{1}{\hbar^2} \left(p^2 - \frac{\varepsilon^2}{c^2} \right) E_0 e^{\frac{i}{\hbar}(px-\varepsilon t)} = 0, \tag{18}$$

which results in the relativistic total energy of a particle with zero rest mass

$$\varepsilon^2 = p^2 c^2. \tag{19}$$

Following de Broglie, we may write the total relativistic energy for a particle with non-zero rest mass

$$\varepsilon^2 = p^2 c^2 + m_0^2 c^4. \tag{20}$$

Inserting expression (20) into eq. (18), it is straightforward from (15) that we get

$$\left(\nabla^2 - \frac{m_0^2 c^2}{\hbar^2}\right)\Psi = \frac{1}{c^2}\frac{\partial^2 \Psi}{\partial t^2}, \tag{21}$$

which is the Klein-Gordon equation [30, 31] for a free particle [29]. Now we want to obtain Schrödinger equation, which is non-relativistic case of eq. (21). The first step is to approximate $\varepsilon^2 = p^2c^2 + m_0^2 c^4$ as follows

$$\varepsilon = m_0 c^2 \sqrt{1 + \frac{p^2}{m_0^2 c^2}} \approx m_0 c^2 + \frac{p^2}{2m_0} \approx m_0 c^2 + \Im\,. \tag{22}$$

After some approximation steps, Ward and Volkmer [29] arrived at the Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\phi = i\hbar\frac{\partial\phi}{\partial t}\,, \tag{23}$$

Where the non-relativistic wave function ϕ is also constrained to the condition that it be normalisable to unit probability.

Hilbert and Batelaan [32] explored equivalence between the quantum and acoustic system. A simple physical system was discussed, which mirrored the quantum mechanical infinite square well with a central delta well potential. They find that the analytic solution to the quantum system exhibits level splitting, as does the acoustic system. They compare the acoustic resonances in a closed tube and the quantum mechanical eigen-frequencies of an infinite square well and showed that the acoustic displacement standing wave is

$$\xi(x) = \xi_{\max}\sin\left(n\pi x/(2a)\right) \tag{24}$$

for the *n*th resonance. Eq. (24) has the same shape as the quantum mechanical wave function.

So we can conclude that there exists formal connection between the classical wave equation and Schrödinger equation, but it still requires some assumptions and approximations. Shpenkov's interpretation of classical wave equation looks as more realistic for atomic and molecular modeling.

TWO ROUTES TO CELLULAR AUTOMATA MODEL OF WAVE EQUATION

A plausible method to describe cellular automata model of wave equation was depicted by Yang and Young [33]. For the 1D linear wave equation, where *c* is the wave speed they presented a scheme

$$\frac{u_i^{n+1}-2u_i^n+u_i^{n-1}}{(\Delta t)^2}=c^2\frac{u_{i+1}^n-2u_i^n+u_{i-1}^n}{(\Delta x)^2}. \tag{25}$$

After some steps eq. (25) can be rewritten in a generic form (by choosing $\Delta t=\Delta x=1,\ t=n$) as follows $u_i^{t+1}+u_i^{t-1}=g(u^t)$, which is reversible under certain conditions. This property comes from the reversibility of the wave equation because it is invariant under the transformation: $t \rightarrow -t$.

O'Reilly has shown that the coupled Maxwell-Dirac electrodynamic system can be implemented in an analog cellular-automaton operating within a 3D regular face-centered cubic lattice [34]. The result of this approach can be expressed in terms of a second order wave equation, namely: $s_i^{t+1}=s_i^t+\dot{s}_i^{t+1}$. He concludes that the second order wave equation is arguably one of the simplest possible continuous-valued cellular automata update equations that do anything physically interesting, though all of electrodynamics can be built of elaborations of this one fundamental interaction.

Thus, cellular approach allows one to construct equations that describe physical systems without using second order equations.

THE TESSELLATTICE AS THE SOURCE FOR THE FORMALISM OF CONVENTIONAL QUANTUM MECHANICAL

A detailed theory of real physical space was developed by Bounias and Krasnoholovets strating from pure mathematical principles (see e.g. Ref. [35]). A submicroscopic theory of physical processes occurring in real physical space was elaborated by Krasnoholovets in a series of works (see e.g., monograph [15]). Those studies show that our ordinary space is constructed as a mathematical lattice of primary topological balls, which was named a tesselllattice. In the tesselllattice, balls play the role of cells. This is a physical vacuum, or aether. Matter emerges at local deformations of the tessellattice when a cell (or some cells) changes its volume following a fractal law of transformations. Such a deformation in the thessellatie can be associated with the physical notion of mass.

The motion of a fractal-deformed cell, i.e. a mass particle, is occurred with the fractal decomposition of its mass owing to its interaction with ongoing cells of the tesselllattice. Such an interaction generates a cloud of a new kind of spatial excitations named 'inertons'. This means that "hidden variables" introduced in the past by de Louis Broglie, David Bohm and Jean-Pierre Vigier have acquired a sense of real quasiparticles of space.

Thus in monograph [15] it has been shown that inertons are carriers of a new physical field (the inerton field), which appears as a basic field of the universe. Inertons as quasi-particles of the inerton field are responsible for quantum mechanical, nuclear and gravitational interactions of matter.

A particle moving in the tessellatice is surrounded with its inerton cloud. This picture can easily be compared with the formalism of quantum mechanics because the particle wrapped with its inerton cloud can be projected to the particle's wave ψ-function determined in an abstract phase space. In such a pattern, the overlapping of ψ-functions of nearest particles means that the particles' inerton clouds overlap and thus we obtain real carriers of the quantum mechanical interaction, which provide a short-range action between the particles studied.

The particle's de Broglie wavelength λ plays the role of a section in which the moving particle emits its inerton cloud and in the next odd section λ these inertons come back to the particle passing on the momentum to it.

How can we write the interaction of a moving particle with its inerton cloud? The interaction can be written between the particle and an ensemble of inertons, which accompany the particle. The ansemble is presented as one intergral object, an inerton cloud. The speed υ_0 of the particle the particle satisfies the inequality $\upsilon_0 << c$. At such presentation, our study is significantly simplified and is reduced to the consideration of a system of two objects: the particle and its cloud of inertons, which the particle periodically emits and adsorbs when moving along its path. In this case the Lagrangian (2.1) is transformed to the following one written in two-dimensional Euclidean space

$$L = \tfrac{1}{2} m_0 \dot{x}^2 + \tfrac{1}{2}\mu_0 \cdot \left[(\dot{\chi}^{\parallel})^2 + (\dot{\chi}^{\perp})^2 \right] - \frac{2\pi}{T}\sqrt{m_0 \mu_0}\, x \dot{\chi}^{\perp} . \tag{26}$$

In the Lagrangian (26)(2.49) the first term describes the kinetic energy of the particle with the mass *m* and the velocity $\dot{x}$, which moves along the axis *X*; the second term depicts the kinetic energy of the whole inerton cloud whose mass is μ_0 and its center-of-mass has the coordinate $\chi^{\parallel}$ along the particle's path and $\chi^{\perp}$ is the transverse coordinate; the third term is the interaction energy between the particle and the inerton cloud where $1/T$ is the frequency of their collisions.

By using the substitution

$$\dot{x}^{\perp} = \dot{\tilde{\chi}} + 2\pi\sqrt{m_0 / \mu_0}\, x / T , \tag{27}$$

we carry out a kind of a canonical transformation that leads to the following

Lagrangian

$$\tilde{L} = \tfrac{1}{2} m_0 \dot{x}^2 - \tfrac{1}{2}(2\pi / T)^2 m_0 x^2 + \tfrac{1}{2}\mu_0 \cdot \left(\dot{\tilde{\chi}}^2 + (\dot{\chi}^{\parallel})^2\right). \tag{28}$$

We can see from the effective Lagrangian (28)(2.51) that in such a presentation the particle's behavior is described as a classical harmonic oscillator and the accompanying inerton cloud moves by its own hidden principle (though it does not disturb the particle).

The Hamiltonian function according to the definition

$$H = \sum_i \dot{Q}_i \partial L / \partial \dot{Q}_i - L .$$

In our case the Hamiltonian is

$$H = \dot{x}\,\partial L / \partial \dot{x} + \dot{\tilde{\chi}}\,\partial L / \partial \dot{\tilde{\chi}} - \tilde{L} . \tag{29}$$

The effective Hamiltonian based on the Lagrangian (28)(2.51) of the oscillating particle in the system of the center-of-mass of the particle and its inerton cloud in the explicit form becomes

$$H = p^2 / (2m_0) + m_0 (2\pi / T)^2 x^2 / 2 . \tag{30}$$

Solutions of the equations of motion given by the Hamiltonian (30) are well known for different presentations. In particular, the function (30) allows one to derive the Hamilton-Jacobi equation

$$(\partial S_1 / \partial x)^2 / (2m_0) + m_0 (2\pi / T)^2 x^2 / 2 = E \tag{31}$$

from which we obtain the equation for a shortened action

$$S_1 = \int_{x_0}^{x} p\, dx = \int_{x_0}^{x} \sqrt{2m_0 [E - (2\pi / T)^2 x^2 / 2]}\, dx . \tag{32}$$

The function (32)(2.55) enables the solution x as a function of t in the form

$$x = \frac{\sqrt{2E / m_0}}{2\pi / T} \sin(2\pi t / T) . \tag{33}$$

Now we can calculate the increment ΔS_1 of the action (32)(2.55) of the particle during the period *T*; in terms of the action-angle variables

$$\Delta S_1 = \oint p\,dx = \oint \sqrt{2m_0\left(E - m_0(2\pi/T)^2 x^2\right)}\,dx$$
$$= \oint \sqrt{2m_0\left(E - E\sin^2(2\pi t/T)\right)}\sqrt{2E/m_0}\cos(2\pi t/T)\,dt$$
$$= 2E\int_0^T \cos^2(2\pi t/T)\,dt = 2E\left(\frac{t}{2} + \frac{\sin(4\pi t/T)}{4(2\pi/T)}\right)\Bigg|_{t=0}^{t=T} = ET. \qquad (34)$$

The final result (34) can be rewritten as follows

$$\Delta S_1 = E\cdot T = E/\nu \qquad (35)$$

where the notation $\nu = 1/T$ is entered.

Since the constant *E* is the initial energy of the particle, i.e., $E = \frac{1}{2}m_0\upsilon_0^2$, the increment of action (35) can also be presented in the form

$$\Delta S_1 = \tfrac{1}{2}m_0\upsilon_0^2\cdot T = m_0\upsilon_0\cdot\tfrac{1}{2}\upsilon_0 T = m_0\upsilon_0\cdot\lambda \qquad (36)$$

where the parameter λ is the spatial amplitude of oscillations of the particle along its path.

If we equate the increment of the action ΔS_1 to the Planck constant h, we immediately arrive at the two major relationships of quantum mechanics introduced by de Broglie for a particle:

$$E = h\nu\,, \qquad \lambda = h/(m_0\upsilon_0)\,. \qquad (37)$$

Thus the amplitude of special oscillation of a particle is exactly the particle's de Broglie wavelength.

Having obtained the relationships (37), we can present the complete action for a particle

$$S = S_1 - Et = \int^x p\,dx - Et \qquad (38)$$

in two equivalent forms:

$$S = m_0\upsilon_0 x - Et \qquad (39)$$

and

$$S = h\cdot(x/\lambda - \nu t)\,. \qquad (40)$$

The relationships (39), (40) and (37) allow the derivation of the Schrödinger equation. If in a conventional wave equation

$$\Delta\psi - \frac{1}{(\upsilon_0/2)^2}\frac{\partial^2\psi}{\partial t^2} = 0 \tag{41}$$

(where $\frac{1}{2}\upsilon_0$ is the average velocity of the particle in the spatial period λ) we insert a wave function, whose phase is based on the action (40),

$$\psi = a\exp\left\{i2\pi[x/\lambda - \nu t]\right\}, \tag{42}$$

and set $\upsilon_0 = \lambda\cdot 2\nu$, we get the wave equation in the following presentation:

$$\Delta\psi + \left(2\pi/\lambda\right)^2\psi = 0\,. \tag{43}$$

Then putting $\lambda = h/p$ and extracting the momentum p from the function (32) (i.e., $p^2 = 2mE$) we finally obtain a conventional time-independent Schrödinger equation

$$\Delta\psi + \frac{2m_0E}{\hbar^2}\psi = 0\,. \tag{44}$$

Thus, we can see that the moving system of a particle and its inerton cloud obyes the Schrödinger equation.

THE PARTICLE'S DEFORMATION COAT AND THE KLEIN-GORDON EQUATION

As we discussed above, Ward and Volkmer [29] demostrated the derivation of the Klein-Gordon equation (21) for a mass particle starting from its total relativistic energy $\varepsilon^2 = p^2c^2 + m_0^2c^4$ (20). They also showed that a non-relatistic approximation of the same energy (20) results in the Schrödinger time-dependent equation (23).

Usually the Klein-Gordon equation [30, 31] is applied for the description of an abstract relativistic particle that does not possess spin. However, the submicrscopic conept of physics presented in monograph [15] makes its possible to relate the Klein-Gordon equation to a real object, namely, a deformation coat that is developed around the mass particle created in the tessellatrice.

In fact the creation a particle means the appearance of a local deformatoion, i.e. a volumetric fractal deformation of the appropriate cell of the tesselllatice. The local deformation must induce a tension state in ambient cells, which may extend only to a definite radius R. So behind the radius R, the tessellattice does not have any distorsion, it is found here in a degenerate state.

The study [15] shows that in the microworld such fundamental physical parameters as mass and charge vary at the motion. Namely, in a section (the even section) equal to the particle's de Broglie wavelength λ the mass m is transferred to a tension ξ and the charge e changes to the magnetic nomopole g. In the odd section λ the mass and charge are restored. The same happened with cells that form the particle's deformation coat. When the particle is moving, it pulls its deformation coat as well, i.e. ambient cells adjust to state of the particle. In the deformation coat the state of cells osciallates between the tension ξ and mass m. A collective oscillating mode of the deformation coat is specified by the energy [15] $E = \hbar\omega$, which in turn equals the total energy of the particle mc^2.

The discussed oscillations can be descrbed by a plane wave mode $E(x,t) = E_0 e^{i(kx-\varepsilon t)}$ (17). Then following the arguments (17) – (21), we immediately derive the Klein-Gordon equation (21). Note in our case the particle that obeys the Klein-Gordon equation is the deformation coat that accompanies the moving particle. This deformation coat is specified with the radius equal to the particle's Compton wavelength [15] (see p. 57).

If the speed υ of a particle satisfies the inequality $\upsilon << c$, we following reasoning (22) and (23) will arrive at the Schrödinger equation (23).

DISCUSSION AND CONCLUSION

We have discussed some weaknesses of the Schrödinger equation for description of atom and molecules. Then we have debated Shpenkov's wave model of atom and molecules based on classical wave equation. His model is able to arrive at a periodic table of elements, which is close to Mendeleyev's periodic law. We also have reviewed a plausible cellular automaton molecular model based on classical wave equation, as an alternative to Cellular automaton quantum mechanics.

At last we have considered the submicrosicopc concept that allows one to easily derive the Schrödinger and Klein-Gordon equations starting from first submicrosopcic principles. It is intetesting that for the first time we now can identify the Klein-Gordon equation with a real object that is described by this equation – it is the particle's deformation coat that is induced in the tessellattice at around the appropriate created canonical particle.

The submicroscopic concept, which is based on space constituted as the tessellattice of primary topological balls, introduces a new physical field, namely the inerton field, which appears as a fundamental field of the universe. Inertons emerge at any motion of particles; in particular, they arise in atoms and around owing to uninterrupted motion of electrons, nuclei and nucleons.

Thus the motion of a quatum system is characterised by its separation to two joined subsystems: the particle itself and its inerton cloud. Their oscillation dynamics exhibits obvious features of the wave motion. Although the deformation coat that accompanies the moving particle behaves in a special way, it is described by the Klein-Gordon equation, which also manifests the wave properties.

Our analysis shows that oscillations of inertons are present in any movement of a material object. Inertons clearly demonstrate wave behaviour. This means that inerton oscillations appear in atoms and molecules. Hence inerton oscillations justify Shpenkov's model [4–14], which applies a classical wave equation of sound to atoms and molecules: the wave funcation Ψ used by Shpenkov describes oscillations of an inerton field and the location of the corresponding nodes in the oscillating wave studied.

Thus, quantum mechanical models, cellular automata, and a cellular automaton molecular model that uses a wave equation can be covered by studies originated from the tessellattice and the submicroscopic behaviour of quantum systems, which invloves an inerton field that binds canonical particles with the tessellattice and between themselves. Further investigations in this direction are recommended, which will shed light on the cornerstones of the microworld.

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REFERENCES

[1] Elze, H.-T. (2015). Are nonlinear discrete cellular automata compatible with quantum mechanics? *J. Phys.: Conference Series 631*, 012069.

[2] Shpenkov, G. P. (2017). Quantum models as classical cellular automata. *J. Phys.: Conf. Series 845*, 012022.

[3] 't Hooft, G. (2014). The cellular automaton interpretation of quantum mechanics, arXiv:1405.1548 [quant-ph].

[4] Shpenkov, G. P., and Kreidik, L. G. (2002). Microwave background radiation of hydrogen atoms, *Revista Ciências Exatas e Naturais 4*(1), 9-18,

http://www.unicentro.br/pesquisa/editora/revistas/exatas/v4n1/Microwave.pdf
[5] Kreidik, L. G., and Shpenkov, G. P. (2002). Important results of analyzing foundations of quantum mechanics, *Galilean Electrodynamics & QED-EAST 13*(2), 23-30, http://shpenkov.janmax.com/QM-Analysis.pdf
[6] Shpenkov, G. P., and Kreidik, L. G. (2004). Dynamic model of elementary particles and fundamental interactions. *Galilean Electrodynamics,* Special Issue – GED East *15*(2), 23-29.
[7] Shpenkov, G. P. (2005). The nodal structure of standing spherical waves and the periodic law: What is in common between them? *Physics Essays 18*(2), 196-206.
[8] Shpenkov, G. P., and Kreidik, L. G. (2005). Schrödinger's error in principle. *Galilean Electrodynamics 16*(3), 51-56, http://shpenkov.janmax.com/Blunders.pdf
[9] Shpenkov, G. P. (2005). The binding energy of helium, carbon, deuterium and tritium in view of shell-nodal atomic model and dynamic model of elementary particles, http://shpenkov.janmax.com/stronginteraction.pdf
[10] Shpenkov, G. P. (2006). An elucidation of the nature of the periodic law, Chapter 7 in The mathematics of the periodic table, Rouvray, D. H. and King, R. B., ed., Nova Science Publishers, New York, pp. 119-160.
[11] Shpenkov, G. P. (2006). A new theory of matter-space-time: Evidences in support of an advantage over the modern theory accepted in physics and the perspective to be of use. A lecture delivered in Military Academy, Warsaw, Poland, October, 20, http://shpenkov.janmax.com/Theory-DM-English.pdf
[12] Shpenkov, G. P. (2007). The shell-nodal structure of the carbon atom and graphene, http://shpenkov.com/pdf/talk2017Berlin.pdf
[13] Shpenkov, G. P. (2010). Anisotropy of unstrained pristine graphene. http://shpenkov.janmax.com/GrapheneAnisotropy.pdf
[14] Shpenkov, G. P. (2013) Dialectical view of the world: The wave model (selected lectures). Vol. I: Philosophical and mathematical background. http://shpenkov.janmax.com/Vol.1.Dialectics.pdf
[15] Krasnoholovets, V. (2017). Structure of space and the submicroscopic deterministic concept of physics. Apple Academic Press, Oakville and Waretown.
[16] Pain, H. J. (2005). The Physics of Vibrations and Waves, 6th ed. J. Wiley & Sons, Ltd. ISBN: 0-470-01295-1(hardback); 0-470-01296-X(paperback).
[17] Rienstra, S.W., and Hirschberg, A. (2014). An introduction to acoustics. Eindhoven University of Technology, www.win.tue.nl/~sjoerdr/papers/boek.pdf
[18] Anonymous, Density Functional Theory for Beginners. (2014). http://newton.ex.ac.uk/research/qsystems/people/coomer/dft_intro.html
[19] Huang, X. (2012). How did Schrödinger obtain the Schrödinger equation?, http://vixra.org/abs/1206.0055

[20] Schrödinger, E. (1926). Quantisation as a Problem of Proper Values. Part I. In Collected papers in Wave Mechanics. Providence, Rhode Island: AMS Chelsea Publishing, http://einstein.drexel.edu/~bob/Quantum_Papers/Schr_1.pdf
[21] Schrödinger, E. (1926). An undulatory theory of the mechanics of atoms and molecules. *Phys. Rev*., Second series *28*(6), 1049-1070.
[22] Anonymous, Atomic Spectra, p. 19-21, http://astrowww.phys.uvic.ca/~tatum/stellatm/atm7.pdf
[23] Fowler, M. (2007). Classical wave equations, p. 10-12. http://galileo.phys.virginia.edu/classes/252/Classical_Waves/Classical_Waves.pdf
[24] Mills, R. L. (2008). The grand unified theory of classical physics. Vol. I. Atomic physics; Vol. II. Molecular physics. Blacklight Power. Printed by Cadmus Communications, A Canveo Company Richmond, UA.
[25] Close, R. A. (2008). The classical wave theory of matter, http://www.classicalmatter.org/ClassicalMatterWaves.html.
[26] Christianto, V. (2014). A review of Schrödinger equation & classical wave equation. *Prespacetime J. 5*(5), http://www.prespacetime.com
[27] Landsman, N. P. (2005). Between classical and quantum, arXiv:quant-ph/0506082.
[28] Rienstra, S.W., and Hirschberg, A. (2014). An introduction to acoustics. Eindhoven University of Technology, www.win.tue.nl/~sjoerdr/papers/boek.pdf
[29] Ward, D. W., and Volkmer, S. (2006). How to derive the Schrödinger equation. arXiv:physics/0610121.
[30] Klein, O. (1926). Quantentheorie und fíunfdimensionale Relativitíatstheorie, Zeit. fíur Phys. 37, 895-906.
[31] Gordon, W. (1926). Der Comptoneffekt nach der Schríodingerschen Theorie, Zeit. fíur Phys. 40, 117-133.
[32] Hilbert, S. A., and Batelaan, H. (2007). Acoustic analog to quantum mechanical level splitting. *Am. J. Phys*. *75*(11), 1003; also in Faculty Publications, Department of Physics and Astronomy, University of Nebraska - Lincoln. Paper 103, http://digitalcommons.unl.edu/physicsfacpub/103.
[33] Yang, X.-S., and Young, Y. (2010). Cellular automata, PDEs, and pattern formation, arXiv: 1003.1983.
[34] O'Reilly, R. C. (2006). An analog cellular automaton model of Maxwell-Dirac electrodynamics. https://pdfs.semanticscholar.org/e214/f1e82db00c12d8266db878f3287691750623.pdf
[35] Bounias, M., and Krasnoholovets, V. (2004). The universe from nothing: A mathematical lattice of empty sets. *International J. Anticipatory Computing Systems 16*, 3-24; arXiv:physics/0309102.